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We discuss the physical effects causing a modification of resonance masses, widths and even shapes in a dilute hadronic gas at late stages of heavy ion collisions. We quantify the conditions at which resonances are produced at RHIC, and found that it happens at $T \approx 120 \text{ MeV}$. Although in the pp case the “kinematic” effects like thermal weighting of the states is sufficient, in AA we see a clear effect of dynamical interaction with matter, both due to a variety of s-channel resonances and due to t-channel scalar exchanges. The particular quantity we focus mostly on is the ρ meson mass, for which these dynamical effects lead to about -50 MeV shift, on top of about -20 MeV of a thermal effect: both agree well with preliminary data from STAR experiment at RHIC. We also predict a complete change of shape of $f_0(600)$ resonance, even by thermal effects alone.

I. INTRODUCTION

The first experimental data from the Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Laboratory have opened a new page in studies of hot/dense hadronic matter. The magnitude of collective effects strongly suggests rapid equilibration in the system. Strong jet quenching, its angular dependence and the absence of compensating jet all suggest the surface emission of jets. Why the bulk of the matter is effectively black, even for $\sim 10 \text{ GeV}$ hadrons/jets, remains unknown at this time. Another unsolved RHIC puzzle is the apparent large fraction of baryons at large p_t . All those phenomena at large transverse momenta are presumably related to properties of extremely hot/dense hadronic matter, preceding formation of the Quark-Gluon Plasma.

In this paper we would however approach the phenomena from the opposite side, focusing instead on the most dilute stages of the collision close to the freezeout. It has been argued over the years that under such conditions hadrons and especially short-lived resonances should be modified, with shifted mass, increased width and even significantly changed shape. The most dilute stage of the collisions, known as *kinetic* freezeout, is much more dilute than nuclear matter. In addition, most of the particles are Goldstone bosons, π, K, η , which interact weakly at low energies. At such low density matter the proposed modifications are expected to be small, although quite observable. They should naturally be studied to set a benchmark, before extrapolation to denser matter close to the QCD phase transition, at which more species of hadrons are actually produced (the chemical freezeout).

Recently the STAR experiment at RHIC reported observed signals of a variety of mesonic resonances such as $\rho, \omega, K^*, f_0(980)$ [1] and ϕ [2] as well as baryonic resonances (which we will not discuss in this work). A data sample reported corresponds to mid-central (40 to 80 percent of the hadronic cross section) AuAu collisions at $\sqrt{s} = 200 \text{ GeV}$ is shown in Fig.1. (They are not yet acceptance corrected, so the numbers extracted from them should not be directly compared to our calculations below.) One can see from comparison of pp and AuAu

spectra that the shape of the $\rho - \omega$ pair of resonances in the $\pi^+\pi^-$ channel is quite different from its classic shape in $e^+e^- \rightarrow \pi^+\pi^-$ or τ decays. Unlike those lepton-production reactions, in which the masses of ρ and ω are nearly degenerate, in pp and even more so in AuAu collisions the ρ mass is shifted downward, while the ω mass stays the same. The (preliminary) fits [1] gave $m_\rho = 0.698 \pm 0.013 \text{ GeV}$ in AuAu and $0.729 \pm 0.006 \text{ GeV}$ in pp, with about unchanged width.

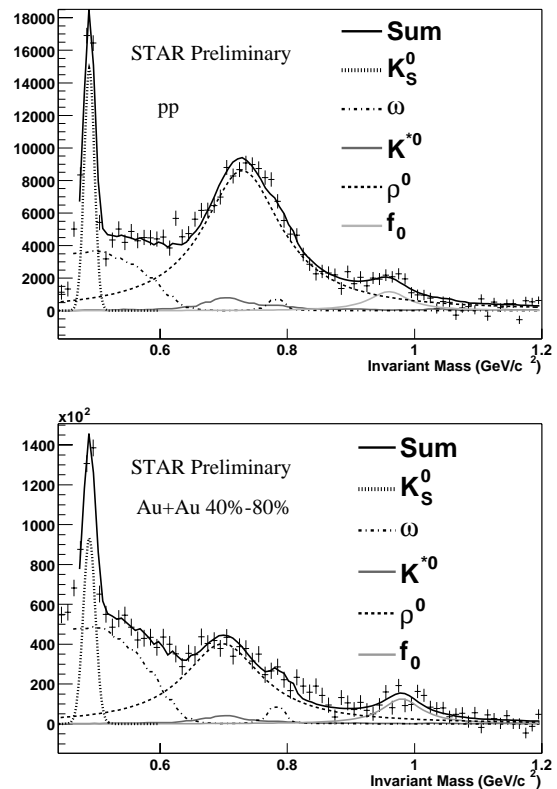


FIG. 1. The invariant mass distributions in pp and mid-central AuAu of the $\pi^+\pi^-$ system, with a transverse momentum cut $0.2 < p_t < 0.9 \text{ GeV}$. The lines indicate contribution of particular resonances according to some model we do not discuss in this work.

In general, resonance modification in hadronic matter results from mutual interaction, and is analogous to the well known phenomena in atomic physics known as spectral line shifts and broadening.

In the first order in density of some a hadronic species n_i , a modification of a hadron of kind j can be expressed in terms of their *forward scattering amplitude* $M_{ij}(t=0, s)$. It is similar to many known phenomena such as e.g. a modification of the photon dispersion law when it propagates through glass. Note that the scattering amplitude is complex, and that this approach gives both the real and imaginary part of the dispersion law modification, also known as the optical potential. Note also that if the dependence on momenta of both particles is strong, appropriate integration with the thermal weights should be performed.

Unfortunately, only partial information about the scattering amplitude can be obtained from the experiment. There are two major theoretical approaches to the issue discussed in literature, to be called an *s-channel* and a *t-channel* one.

The former approach assumes that the scattering amplitude is dominated by *s-channel* resonances which are known to decay into the $i+j$ channel. For most mesons such as π, ω, ρ, K in a gas made of pions such calculation has been made [5–8]. It is usually true that the single most important part of scattering amplitude is the lowest resonance. A general rationale for that is that higher resonances typically decay into states with higher multiplicity, with the 2-body channel in question having a small branching ratio. In the case we will discuss most below – the $\rho\pi$ scattering – it is the axial resonance a_1 . Generic diagrams for such processes look like that depicted in Fig.2(a): and in dilute matter the modification of the ρ is simply proportional to the appropriately weighted pion density (see below).

Such an approach is of course rather limited, applicable only for dilute matter. Indeed, if one would like to follow it to second order in density with a double collision Fig.2(c), one should be able to extrapolate the scattering data off-shell (an intermediate line with an asterisk). The same problem appears if one may think about loop corrections. Furthermore, generic 3-body collisions cannot be accessed experimentally, only their part due to intermediate resonances with the known 3-body decays (see Fig.2(d)) can.

The predicted [5] ρ mass shift due to the a_1 in an equilibrated* pion gas was found to be small, not exceeding -10 MeV. The sign of this effect can be explained

*That is, with zero pion chemical potential. The calculations below correspond to significant μ_π which of course increase the effect.

as follows: states of the same quantum numbers repel each other if mixed, and thermally populated $\rho\pi$ states have total energy mostly below the mass of a_1 .

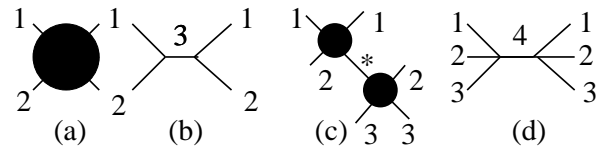


FIG. 2. Forward scattering amplitude for particles of type 1 and 2 (a) can be approximated by intermediate resonance 3 (b). Even the double 2-body scatterings (c) include an off-shell particle (asterisk) which cannot be calculated without assumptions. An exceptional case is a single resonance 4 which has an observable 3-body decay (d).

Similar effects due to scattering on nucleons have been studied e.g. by Rapp, Wambach and collaborators, see [9] and references therein. There is however an important difference in sign of the effect. The major resonance relevant for ρN scattering on nucleons, $N^*(1520)$, is not only below the thermally populated states but even below $m_\rho + m_N$: thus it pushes the ρ mass upward. The lower excitation, which is the $N^*(1520)N^{-1}$ state, is at the same time pushed down. Another important effect discussed a lot in these works is significant broadening of both states, leading to a complicated quasi-continuum spectrum in place of two well separated states.

Let us now turn to the second *t-channel* approach, attributing the mass shifts to mutual attractive interaction between hadrons, such as *t-channel* exchange of a scalar isoscalar σ meson, see Fig.3. We first comment on a significant controversy related to its association with the issue of depletion of the chiral condensate and chiral restoration phenomenon. Let us try to explain briefly what is its status at this point.

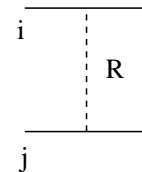


FIG. 3. Forward scattering amplitude for particles of type i and j exchanging the “radial” scalar R .

On the one hand, a modification of the quark condensate can be calculated at small temperatures from general chiral theory [14] without a problem, the result is

$$\langle \bar{q}q \rangle(T) = \langle \bar{q}q \rangle(0) \left(1 - \frac{T^2}{8f_\pi^2} - \frac{T^4}{384f_\pi^4} + \dots \right) \quad (1)$$

The hadronic masses appear due to chiral symmetry breaking, and in models like NJL it happens via the so called constituent quark mass which is directly proportional to the quark condensate. Therefore naively

$$\frac{m_\rho^*}{m_\rho} \sim \frac{\langle \bar{q}q \rangle^*}{\langle \bar{q}q \rangle} \quad (2)$$

which is sometimes referred to in the literature as a “Nambu scaling”.

Here and below in-matter quantities are marked by the asterisk. Brown and Rho have proposed another scaling (see the latest work [10] for earlier references and explanations)

$$\frac{m_\rho^*}{m_\rho} \sim \frac{f_\pi^*}{f_\pi} \sim \left(\frac{\langle \bar{q}q \rangle^*}{\langle \bar{q}q \rangle} \right)^\alpha \quad (3)$$

with $\alpha = 1/2$ for low densities,

The problem however is that such relations are not respected by low-T chiral calculations. In particular, the $O(T^2/f_\pi^2)$ effect which is present in the quark condensate (1) as well as in of the vector and axial correlation functions [15], is absent in physical effects like the total energy of the pion gas [13] or in the mass shifts, which appears only in order $O(T^4)$, see e.g. [16]. More generally, it is clear that a t-channel exchange of a sigma meson between a pion (with a *non*-derivative coupling) and any other hadron would violate the Goldstone theorem and lead to a non-zero pion mass. This contribution must therefore cancel out with some other diagrams, as seen in explicit calculations in many models.

Predictions for next order $O(T^4)$ mass shifts are model-dependent and there is no theoretical consensus even about the *sign* of the contribution. The $O(T^4)$ calculation performed in [16] has predicted that both ρ, a_1 masses decrease slightly, in equilibrium $T = 150 \text{ MeV}$ was found to be $\delta m_\rho^{EI} = -6.4 \text{ MeV}$.

In contrast to that, Pisarski [17] has built a set of simple effective models incorporating π, ρ, a_1 interactions (which all of course respect chiral symmetry) and concluded that no prediction can be really made. In one of them, enforcing VDM at any T, the result is that the ρ mass should *grow* with T significantly, joining a_1 at at chiral restoration point at the following intermediate mass

$$m_\rho^2(T_\xi) = m_{a_1}^2(T_\xi) = \frac{2}{3}m_\rho^2(0) + \frac{1}{3}m_{a_1}^2(0) = (962 \text{ MeV})^2 \quad (4)$$

In contrast to scalar exchanges with pions, such exchanges between other hadrons definitely exist. (For nucleons those are in fact responsible for a large portion of nuclear forces and binding.) The effect of nucleon density on quark condensate related with the nucleon sigma term [18] is large even at nuclear matter density

$$\frac{\langle \bar{q}q \rangle^*}{\langle \bar{q}q \rangle} = 1 - \frac{\Sigma_{\pi N} n}{f_\pi^2 m_\pi^2} + \dots \approx 1 - .36 \frac{n}{n_0} \quad (5)$$

where n_0 is the nuclear matter density. The resulting BR scaling prediction

$$m_\rho^* = m_\rho (1 - 0.36 \frac{n}{n_0} + \dots)^{1/2} \quad (6)$$

leads to good agreement with experimental information on scattering on nuclei.

This apparent contradiction is discussed in multiple papers. We would follow [11] where it has been especially clearly emphasized that one may think of *two* different scalar fields which can be exchanged. One is the old σ of the linear sigma-model, a chiral partner of the pion. The other we will call R [†]

$$R^2 = \sigma^2 + \pi^2 \quad (7)$$

The main difference between VEVs $\langle \sigma \rangle$ and $\langle R \rangle$ is that the former gets the pion loop $O(T^2)$ correction while the latter does not since in the excitations along the chiral circle the radius does not change. In a different language, the interaction Lagrangian describing how R interacts with pions is different from a sigma model one, it has a derivative coupling like for any other particle. Therefore R-exchanges with pions does not give them a mass and does not violate the Goldstone theorem; there is no need for cancellations. The mass shifts for N and ρ can then in the first order be written as R-exchange diagram or [11]

$$\delta M_N = \frac{3}{2} \delta M_\rho = -\frac{g_0^2 n}{m_R^2} \quad (8)$$

In this simple picture $g_0 = M_n/f_\pi \approx 10$ and R can be viewed as the Walecka scalar meson, see also [12] where R is denoted as χ .

B. Resonance mass modification: experiments

We will not go into long history of detailed ρ -meson shape, but simply state that the difference between its appearance in elementary reactions ($e^+e^- \rightarrow \pi^+\pi^-$ or $\tau \rightarrow \nu_\tau \pi^+\pi^-$ decays) and hadroproduction reactions is by now well known. The latest Review of Particle Properties (RPP) [19] averages the rho mass for these two classes of experiments separately, with a clear systematic difference of the order of 10 MeV between them:

$$m_\rho^{\text{leptoproducted}} = 775.9 \pm .5 \text{ MeV}, \quad (9)$$

$$m_\rho^{\text{hadroproduced}} = 766.5 \pm 1.1 \text{ MeV} \quad (10)$$

. We return to discussion of this effect in section IIB below.

A ρ mass shift $\delta m_\rho = 160 \pm 35 \text{ MeV}$ observed in nuclear experiments has been reported by Lolos et al [3] in

[†]From the radius - in [11] it was called θ which may be mistakenly taken as an angle or may be confused with the widely used θ field, the U(1) pseudoscalar. R is the χ of Brown and Rho [12].

the ${}^3\text{He}(\gamma, \rho^0)ppn$ reaction. An even larger shift has been reported by Huber et al [4] when even lower γ energies were used. Although ${}^3\text{He}$ is not a dense nucleus, with average density of about half of nuclear matter, the kinematic of this experiment is sub-threshold for one nucleon. It was therefore argued that the emerged ρ is actually in the vicinity of at least two nucleons, so that the local density is large.

Heavy ion dilepton experiments at SPS, such as HELIOS3, NA38 and especially CERES have indeed found that the vector spectral density of hot/dense matter is very different from that observed in elementary pp collisions. While in the latter case there exist a clear gap between ρ, ω mesons and low-mass pairs originated from Dalitz decays of η and other light mesons, no such gap is present in heavy ion collisions and a continuum of excited states is seen down to the invariant mass of $\sim 400 \text{ MeV}$. Although these results agree with various calculations such as the Rapp-Wambach ones mentioned above, these dilepton data are still very limited statistically and have very crude mass resolution which does not make it possible to see the ρ modification or even separate it from the ω or even the $N^*(1520)N^{-1}$.

The situation is very different for STAR experiment we mentioned in the Introduction, in which the shifts are smaller and the width apparently is not growing, while the invariant mass resolution is excellent and all resonances are seen as clearly resolved separate peaks.

II. MODIFICATION OF ρ MESON AT RHIC

A. Kinetic freezeout at RHIC

Early papers on statistical and hydrodynamical models of multiparticle production had assumed complete equilibrium matter until the final freezeout. Clear separation between two different stages, the chemical (at T_{ch}) and kinetic (at T_k) freezeouts has been made in the mid-1990's. Basically it reflects very different rates of elastic and inelastic reactions at low energies dominating rescattering of secondaries.

In particular, in ref. [21] the following argument has been put forward: at AGS/SPS energies most of the observed velocity of collective radial flow can *only* develop in between the two, at $T_{ch} < T < T_k$.

In that paper also the following natural definition of the kinetic freezeout condition has been given: a secondary (say a pion) emitted at this time has equal chances to be either rescattered or escape:

$$P_{\text{escape}} = P_{\text{rescattering}} = \frac{1}{2} \quad (11)$$

which we will be using below.

At the particular conditions we are interested in, at RHIC $\sqrt{s} = 200 \text{ GeV}$, the numerical values of the parameters are as follows. The rapidity density dN/dy at

mid-rapidity of central AuAu collisions for relevant particles are about 300 for each species of pions, 29 protons and 22 antiprotons. These numbers have large feed-down from resonance decays, which should be accounted for when their ratios are used to derive the chemical freezeout conditions. From [1] it was determined that those are about

$$T_{ch} = 170 \text{ MeV} \quad \mu_b = 28 \text{ MeV} \quad (12)$$

By definition of chemical equilibrium, all other chemical potentials for non-conserved charges are zero at this stage.

After inelastic reactions are frozen and only elastic rescatterings take place, the number of particles of each species are separately conserved. Therefore chemical potentials for all species μ_i become *non-zero* at the hadronic stage, see details in refs [22,21,23,24]. The only important numbers for our discussion below are the value of the kinetic freezeout temperature and the values of chemical potentials for pions and nucleons. Assuming entropy conservation (adiabatic expansion) and natural conservation of baryons one can determine all parameters along a particular cooling path, provided one point on it is known. Using (12) one gets it, and using either kinetics, or hydro-based fit to data, one finds that the kinetic freezeout at RHIC happens at

$$\begin{aligned} T_k &\approx 100 \text{ MeV}, \quad \mu_\pi \approx 81 \text{ MeV} \\ \mu_N &\approx 380 \text{ MeV} \quad \mu_K \approx 167 \text{ MeV} \end{aligned} \quad (13)$$

This translates into the following densities of the pions (all 3 of them)

$$n_\pi \approx 0.06 \text{ fm}^{-3} \quad (14)$$

With $\bar{p}/p \approx 0.75$ we get the following density of all nucleons plus anti-nucleons together

$$n_{N+\bar{N}} \approx 0.0075 \text{ fm}^{-3} \quad (15)$$

or only 1/20 of the nuclear matter density. Accounting for all baryonic resonances will increase them by nearly a factor 3 and bring them in agreement with measured final yields mentioned above.

The next issue we discuss is how the freezeout conditions for most particles (pions) relate to those for *resonances*. When those are actually observed as peaks in the invariant mass distribution of several secondaries, it is only possible when none of the secondaries have been rescattered. Suppose for simplicity the resonance is produced and decays into the same mode with N identical secondaries of type i (N=2 for $\rho \rightarrow 2\pi$). Its observable production at time t is proportional to the following combination of the density needed for production and the probability that all secondaries do escape, namely

$$P(t) \sim \left[n_i(t) \exp\left(-\int_{t+\tau}^{\infty} dt' \sum_j n_j(t') <\sigma_{ij} v_{ij}>\right) \right]^N \quad (16)$$

where $n_i(t)$ is time-dependent density of relevant secondary in matter, and $\langle \sigma_{ij} v_{ij} \rangle$ are thermally averaged cross section and velocity for scattering on type-j particle and τ is the resonance lifetime.

This expression can be simplified significantly, provided the lifetime is ignored, and the matter expansion is parametrized by a single power behavior $n_i(t) \sim 1/t^a$. The cross section is approximately time-independent and can be eliminated in favor of the freezeout time, using its definition (11), leading to

$$P_{resonance}(t) \sim \left[t^{-a} \exp(-\ln(2) \left(\frac{t_k}{t}\right)^{a-1}) \right]^N \quad (17)$$

which for any N has the maximum at

$$t_{res} = t_k [\ln(2) * (1 - 1/a)]^{\frac{1}{a-1}} \quad (18)$$

As hydro simulations show[‡] the value of the index is actually not constant over time; it changes from the Bjorken $a=1$ at early time of 1d expansion to a value $a \approx 2.5$ at relevant times $t \sim 10 fm/c$, not quite reaching ultimate 3 for 3d expansion. For this index, $t_{res} \approx 0.6t_k$. The distribution (17) for N=2 is plotted in Fig.4, and one can see that the peak is rather sharp.

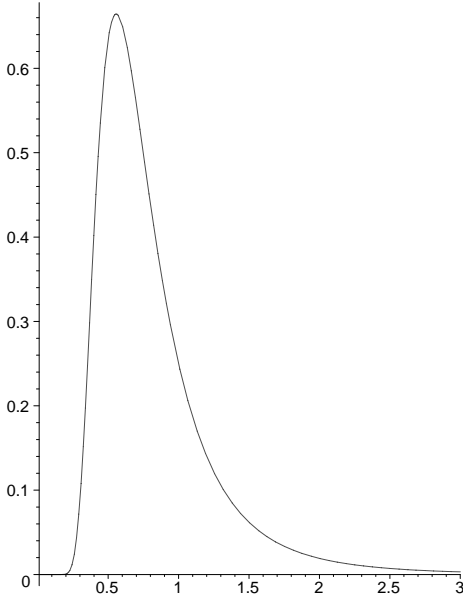


FIG. 4. Probability distribution over the 2-pion resonance production time t , in units of pion freezeout time t_k .

Finally, one can deduce from this time the relevant temperature using a power-fit for density at the hadronic stage $n \sim T^5$ which gives

$$\frac{T_{res}}{T_k} = \left(\frac{t_{res}}{t_k} \right)^{-a/5} \approx 1.2 \quad (19)$$

In effect, we found that the observed resonances are produced somewhat earlier and a bit higher temperature T_{res} compared to the kinetic production time, for all N. The conditions on the same adiabatic path at such temperature are:

$$\begin{aligned} T_{res} &\approx 120 MeV, \quad \mu_\pi \approx 62 MeV \\ \mu_N &\approx 270 MeV \quad \mu_K \approx 115 MeV \end{aligned} \quad (20)$$

with densities about 1.4 higher than at the kinetic freeze-out.

B. Elementary effects due to the heat bath

Processes like $\pi\pi$ scattering in an equilibrium heat bath can be represented as follows

$$dW = [f(p_1)f(p_1)] |M|^2 d\Gamma [(1 \pm f(p_3))(1 \pm f(p_4))] \quad (21)$$

where (i) the first two factors are thermal occupation factors for the incoming particles; (ii) $|M|$ is the matrix element of the re-scattering, (iii) is the phase space and finally (iv) represent final state corrections for induced radiation, \pm for final bosons/fermions; (iii) In dilute matter one can presumably use unmodified matrix element and the phase space.

The so-called “elementary” hadronic reactions like pp collisions at high energies are in fact very complex phenomena, which are far from being understood. Unlike heavy ion collisions, those do not show collective phenomena like flow and one may think they certainly do not contain equilibrated hadronic matter. And yet, as noticed back in 1960’s by Hagedorn and others, the particle composition in pp and even $e + e-$ collisions can be well reproduced by statistical models, see e.g. recent comparison [20]. The fitted temperature is about the same as the chemical freezeout temperature T_{ch} mentioned above. Two- and even many-body distributions follow thermal distributions very well, as data showed already in 1970’s. So it is natural[§] to apply it to 2 pions which are forming the ρ meson in question as well.

Now, if two initial pions come from the heat bath even in pp, the initial thermal factors should be present. At the time the resonance decays, on the other hand, the secondaries making that heat bath are already sufficiently far away. From this it follows that (i) the resonance decay happens in vacuum, its matrix element is unmodified, and (ii) the two final state induced radiation factors are absent.

[‡]We thank P.Kolb who provided plots from which the index value was estimated.

[§]The authors thank P.Braun-Munzinger who suggested this idea to them.

The Boltzmann factor is of course a function of the energy in the matter rest frame, not in the resonance rest frame. However as a simple first approximation one may neglect the thermal motion of rather heavy resonances and assume them to be at rest in the heat bath. This simply combines the Boltzmann factor $\exp(-M/T)$ containing the invariant mass $M^2 = (p_+ + p_-)^2$ with the (appropriately modified for p-waves if needed) Breit-Wigner matrix element.

The position of the maximum can then be obtained in a simple analytic way, provided the resonance width is large enough. Approximating the resonance by

$$\frac{\exp(-M/T)}{1 + (4/\Gamma^2)(M - m_{res})^2} \approx \quad (22)$$

$$\exp[-(\frac{4}{\Gamma^2})(M^2 - 2Mm_{res} - M^2 + \frac{2M\Gamma^2}{8T})]$$

one gets the mass shift due to (sufficiently large) temperature T to be approximately

$$\delta m_{max} \approx -\frac{\Gamma^2}{8T} \quad (23)$$

For ρ emitted at $T = 120 \text{ MeV}$ this expression gives the shift - 23 MeV. This is to be compared to -10 MeV in average low energy hadroproduction reactions (10) and to a shift of about -30 MeV in preliminary STAR pp data [1] mention in the Introduction.

The important point here is that this effect is *quadratic* in width. For example, the 3 times more narrow K^* is predicted to be shifted by this effect an order of magnitude less. In contrast to that, much wider resonances – e.g. the famous σ -meson, now listed as $f_0(600)$ [19] with a width of about 300 MeV – are changed beyond recognition. Applying the same expression we see that in the pp case a Boltzmann factor at $T \sim 160 \text{ MeV}$ transform it into a very wide structure (which is indeed seen in hadroproduction), while at kinetic freezeout of heavy ion conditions $T \sim 100 \text{ MeV}$ it peaks at very low masses instead, see Fig.5(b). (Since it is an s-wave, not p-wave resonance as ρ , its threshold suppression is weaker.)

We argued above that in the pp case the 2 pions forming the ρ come from a heat bath, while the outgoing pions propagate essentially in free space. In contrast to that, in heavy ion collisions the local expansion rates are slow compared to the ρ lifetime, and therefore it decays in matter. So, in principle one should include the final state factor $(1 \pm f)^2$. Direct experimental access to the magnitude of thermal occupation rate at freezeout is provided by the integrated HBT correlator. For ρ it is however a small effect modifying the width by about 10 percent and not affecting the mass. It is larger for ω : see discussion of it in [5].

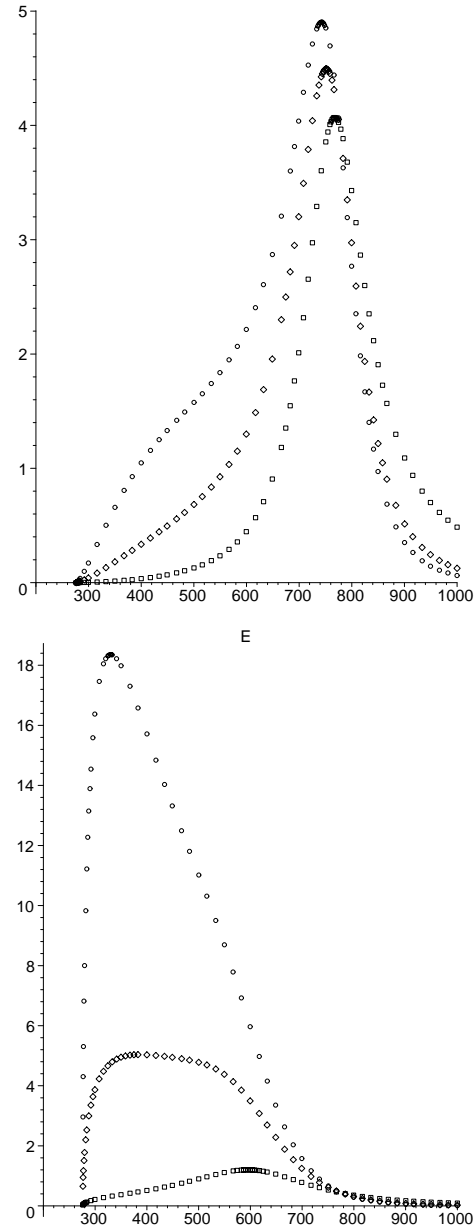


FIG. 5. The shape of the ρ (a) and $f_0(600)$ (b) as they appear in elementary reactions (squares), modified by a Boltzmann factor at chemical freezeout $T=165 \text{ MeV}$ (diamonds) and at chemical freezeout (circles)

C. Modification due to rescattering on pions, kaons and nucleons

The formulae are all well known [5], for the record we use the resonance 2-pion formfactors in the vector and scalar channels, which include the phase space and p^2 in the p-wave matrix element in the width

$$FF_{sq} := \frac{m^4}{(m^2 - E^2 + f)^2 + m^2 \Gamma_v^2} \quad (24)$$

$$FF_{sigma} := \frac{m^4}{(m^2 - E^2 + f)^2 + m^2 \Gamma_\sigma^2} \quad (25)$$

where

$$\Gamma_v := \frac{\Gamma m k^3}{E k v^3} \quad (26)$$

$$\Gamma_\sigma := \frac{\Gamma m k}{E k v} \quad (27)$$

$$k := \frac{1}{2} \sqrt{E^2 - 4 m_\pi^2} \quad (28)$$

$$kv := \frac{1}{2} \sqrt{m^2 - 4 m_\pi^2} \quad (29)$$

We found that the resonance mass shifts due to s-channel resonances depend rather weakly on the emission time, making predictions rather stable. In particular, the difference between (20) and (13) is at the level of 10-15 percent, much less than density difference which is 40 percent. This is explained by a specific shape of the real part, with negative and positive contributions weighted with a thermal weight. At lower temperature higher states contribute less and their cancellation is less pronounced.

As seen from the Table, the interaction of ρ with the pions lead to negative shift of the order of -30 MeV, while with kaons it is basically zero.

Due to $N^*(1520)$ the interaction with nucleons leads to positive shift. There are however resonances above the threshold which are not included; those will somewhat reduced our estimate. On the other hand, due to $S=I=3/2$ of the Δ state, it has rather large statistical factor and significant population at the relevant stage. There are sub-threshold $\rho - \Delta$ resonances in the 2 GeV mass region and thus one can expect an effect similar in sign and magnitude to those from $N^*(1520)$, further increasing the upward shift.

An opportunity to test the role of $N\rho$ interaction is provided by heavy ion collisions at lower collision energies, of the order \sqrt{s} few GeV/N (like expected in new GSI proposed heavy ion project). Since this increases the baryon fraction by an order of magnitude, the combined effect of $\rho - N$, $\rho - \Delta$ resonances can compensate or even overcome the negative shifts discussed above. The measurement of the exact mass and shift of ρ in $\pi\pi$ mode at such conditions is therefore of significant interest.

TABLE I. A set of resonances considered

Name/Mass	Width	Branching	Mass Shift
$a_1(1260)$	400	0.6	-19
$a_2(1320)$	104	0.7	-15
$K_1(1270)$	90	0.4	+1.6
$K_2(1430)$	100	0.087	-0.4
$N^*(1520)$	370	0.2	10

D. The effect of mutual attraction

As explained above, the majority of the particles in the matter are Goldstone bosons π, K, η which do not produce attraction due to R-exchanges. There should however be an effect due to other particles, such as vector mesons and baryons. Unlike the s-channel resonances discussed above, the t-channel attraction scales as density, and thus is 40 percent stronger at (20) as compared to (13). Using a simple expression for the mass shift (8) and the R-scalar mass $M_R \approx 800 \text{ MeV}$ one gets

$$\delta m_\rho^N \approx -28 \text{ MeV} \quad (30)$$

(in agreement with that from the BR scaling) due to all $\bar{B} + B$. An additional shift of the magnitude of

$$\delta m_\rho^v \approx -10 \text{ MeV} \quad (31)$$

comes from scalar R-exchanges between ρ and vector mesons ρ, ω, K^* .

III. DISCUSSION AND OUTLOOK

Summarizing the paper, we have found that: (i) a simple initial state occupation effect is sufficient to explain the ρ mass shift between pp and leptonproduction; (ii) the effect is predicted to be quadratic in width, so it should be an order of magnitude smaller for K^* . At the same time it leads to very significant shape deformation for the sigma $f_0(600)$ pushing it downward toward the threshold; (iii) we found that all resonances are emitted at some time between chemical and kinetic freezeout, at $T \approx 120 \text{ MeV}$; (iv) dynamical effects due to interaction of the ρ with surrounding matter lead to about -50 MeV additional ρ mass shift, relative to a shift in pp mentioned above.

All these conclusions about the mass shift of the ρ agree with the first STAR data shown at Quark Matter 02 [1]. Hopefully quantitative results, with fitted values of mass and width versus centrality, will be soon available for more quantitative comparison.

The main uncertainties of our calculations of s-channel resonances come from poorly known branching ratios for modes containing the ρ mesons we focus on. It is clear that smaller contributions of many more resonances are missing, which may or may not average out to zero. We think the accuracy of our predictions are therefore at the level of 20-30 percent. Observation of similar phenomena with several resonances and at variable conditions (such as N/π ratio) would help to reduce this uncertainty. Very interesting would be precise measurements of the line shape of two more vector mesons, K^* and ϕ , and possibly a shape of ω in 3 pion channel.

In this paper we have not addressed the issue of the ρ width and its broadening: however as data would become more accurate, it would provide an opportunity to

better test the relative role of s-channel and t-channel contributions to the mass shift, since the latter are not associated with the broadening and former produce well defined predictions.

STAR data also display a peak in $\pi\pi$ channel below ρ , at invariant mass $M \sim 600 \text{ MeV}$. It can be partly $\sigma = f_0(600)$, which is however predicted in this work to be deformed and move downward. It is likely to be $2/3$ of the ω with the π^0 missing, or a trace of baryonic resonance like $N^*(1520)$.

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